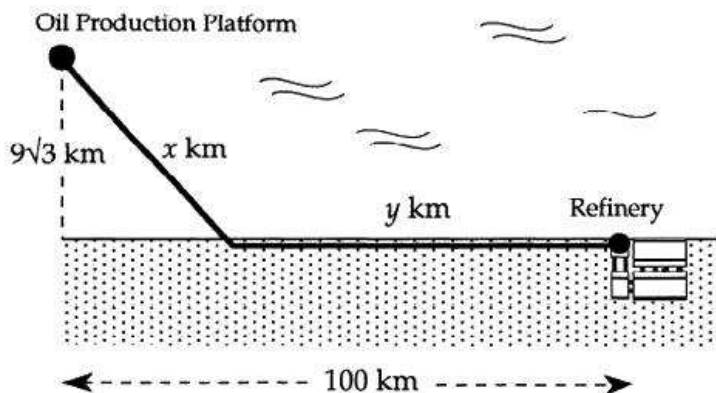


differentiate chain rule

[SQA]

1. An oil production platform, $9\sqrt{3}$ km offshore, is to be connected by a pipeline to a refinery on shore, 100 km down the coast from the platform as shown in the diagram.



The length of underwater pipeline is x km and the length of pipeline on land is y km. It costs £2 million to lay each kilometre of pipeline underwater and £1 million to lay each kilometre of pipeline on land.

- (a) Show that the total cost of this pipeline is £ $C(x)$ million where

$$C(x) = 2x + 100 - \left(x^2 - 243\right)^{\frac{1}{2}}. \quad (3)$$

- (b) Show that $x = 18$ gives a minimum cost for this pipeline.

Find this minimum cost and the corresponding total length of the pipeline. (7)

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(a)	1	C	NC	A6		1993 P2 Q11
(a)	2	A/B	NC	A6		
(b)	1	C	NC	C11, C21		
(b)	6	A/B	NC	C11, C21		

(a) •¹ $C = 2x + y$

•² $\sqrt{x^2 - (9\sqrt{3})^2}$

•³ for completing proof

(b) •⁴ knowing to differentiate

•⁵ $\frac{1}{2}(x^2 - 243)^{-\frac{1}{2}}$

•⁶ $\times 2x$

•⁷ $C'(18) = 0$

•⁸ justification of minimum e.g. nature table

•⁹ $C = 127$

•¹⁰ $x + y = 109$

	18 ⁻	18	18 ⁺
$C'(x)$	-	0	+
	\	—	/
	minimum		

[SQA]

2. Given that $f(x) = (5x - 4)^{\frac{1}{2}}$, evaluate $f'(4)$.

3

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	1	C	CN	C21	$\frac{5}{8}$	2000 P2 Q8
	2	A/B	CN	C21		
<ul style="list-style-type: none">•¹ pd: differentiate power•² pd: differentiate 2nd function•³ pd: evaluate $f'(x)$					<ul style="list-style-type: none">•¹ $\frac{1}{2}(5x - 4)^{-\frac{1}{2}}$•² $\times 5$•³ $f'(4) = \frac{5}{8}$	

[SQA]

3. (a) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$.

2

(b) Hence write down $\int \frac{x^2}{(8-x^3)^{\frac{1}{2}}} dx$.

1

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(a)	2	A/B	CN	C21	$-\frac{3}{2}x^2(8-x^3)^{-\frac{1}{2}}$	2002 P1 Q10
(b)	1	A/B	CN	C24	$-\frac{2}{3}(8-x^3)^{\frac{1}{2}} + c$	

- ¹ pd: process differentiation
- ² pd: use the chain rule
- ³ ic: interpret answer from (a)

- ¹ $\frac{1}{2}(8-x^3)^{-\frac{1}{2}}$
- ² $\dots \times -3x^2$
- ³ $-\frac{2}{3}f(x)$ **or** $-\frac{2}{3}(8-x^3)^{\frac{1}{2}}$

[SQA]

4. Given that $f(x) = 5(7 - 2x)^3$, find the value of $f'(4)$.

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	NC	C21		1991 P1 Q13
$\bullet^1 (7 - 2x)^2$ $\bullet^2 \times 15$ $\bullet^3 \times -2$ $\bullet^4 -30$						

[SQA] 5. Differentiate $2x^{\frac{3}{2}} + \sin^2 x$ with respect to x . 4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	1	C	NC	C21		1992 P1 Q11
	3	A/B	NC	C21		

•¹ $3x^{\frac{1}{2}}$

•² $(\sin x)^2$ stated or implied by •³

•³ $2\sin x$

•⁴ $\times \cos x$

[SQA] 6. Find the derivative, with respect to x , of $\frac{1}{x^3} + \cos 3x$. 4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	NC	C21		1994 P1 Q10

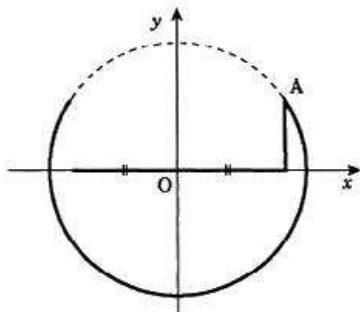
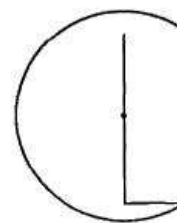
•¹ x^{-3} stated or implied by •²

•² $-3x^{-4}$

•³ $-\sin 3x$

•⁴ $\times 3$

7. Linktown Church is considering designs for a logo for their Parish magazine. The 'C' is part of a circle and the centre of the circle is the mid-point of the vertical arm of the 'L'. Since the 'L' is clearly smaller than the 'C', the designer wishes to ensure that the total length of the arms of the 'L' is as long as possible.



The designer decides to call the point where the 'L' and 'C' meet A and chooses to draw co-ordinate axes so that A is in the first quadrant. With axes as shown, the equation of the circle is $x^2 + y^2 = 20$.

- (a) If A has co-ordinates (x, y) , show that the total length T of the arms of the 'L' is given by $T = 2x + \sqrt{20 - x^2}$. (1)
- (b) Show that for a stationary value of T , x satisfies the equation $x = 2\sqrt{20 - x^2}$. (5)
- (c) By squaring both sides, solve this equation. Hence find the greatest length of the arms of the 'L'. (3)

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(a)	1	A/B	CN	CGD		1995 P2 Q11
(b)	1	C	CN	C21		
(b)	4	A/B	CN	C21		
(c)	1	C	CN	C11		
(c)	2	A/B	CN	C11		

(a) •¹ $T = x + x + y$ and $y^2 = 20 - x^2$

(b) •² appearance of $\frac{dT}{dx} = 2 + \dots$

•³ $\frac{1}{2}(20 - x^2)^{-\frac{1}{2}}$

•⁴ $\times -2x$

•⁵ $\frac{dT}{dx} = 0$ stated or implied

•⁶ completing proof

(c) •⁷ $x^2 = 4(20 - x^2)$

•⁸ $x = 4$ (accept $x = \pm 4$)

•⁹ justifying $x = 4$ gives $T_{\max} = 10$

[SQA] 8. If $f(x) = \cos^2 x - \frac{2}{3x^2}$, find $f'(x)$.

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	C	NC	C21, C1		1990 P1 Q19
	2	A/B	NC	C21, C1		

•¹ $-\frac{2}{3}x^{-2}$

•² $2\cos x$

•³ $\times(-\sin x)$

•⁴ $\frac{4}{3}x^{-3}$

[SQA] 9. Differentiate $4\sqrt{x} + 3 \cos 2x$ with respect to x .

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	C	NC	C21, C1		1993 P1 Q9
	2	A/B	NC	C21, C1		

•¹ $4x^{\frac{1}{2}}$

•² $2x^{-\frac{1}{2}}$

•³ $-\sin 2x$

•⁴ $\times 2$

[SQA] 10. Find $\frac{dy}{dx}$ given that $y = \sqrt{1 + \cos x}$.

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	3	A/B	NC	C21, C20		1996 P1 Q13

$\bullet^1 (1 + \cos x)^{\frac{1}{2}}$ stated or implied by \bullet^2
 $\bullet^2 \frac{1}{2}(1 + \cos x)^{-\frac{1}{2}}$
 $\bullet^3 x - \sin x$

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	3	A/B	NC	C21, C20, T2		1998 P1 Q16
<div><div><div><div>•¹</div><div>$2(\sin x + 1)$</div></div><div><div>•²</div><div>$\times \cos x$</div></div><div><div>•³</div><div>$\frac{3\sqrt{3}}{2}$</div></div></div><div>Alternative</div><div><div><div>•¹</div><div>expand and differentiate $2 \sin x + 1$</div></div><div><div>•²</div><div>differentiate $\sin^2 x$</div></div><div><div>•³</div><div>$\frac{3\sqrt{3}}{2}$</div></div></div></div>						

[END OF QUESTIONS]